Journal of Criminal Law and Criminology

Volume 61 | Issue 3 Article 12

1971

Mathematical Model for Bullet Ricochet

Mohan Jauhari

Recommended Citation

Mohan Jauhari, Mathematical Model for Bullet Ricochet, 61 J. Crim. L. Criminology & Police Sci. 469 (1970)

THE JOURNAL OF CRIMINAL LAW, CRIMINOLOGY AND POLICE SCIENCE
Vol. 61, No. 3

MATHEMATICAL MODEL FOR BULLET RICOCHET*

MOHAN JAUHARI

Mohan Jauhari, M.Sc. is Assistant Director, Central Forensic Science Laboratory, Government of India, Calcutta, India. Mr. Jauhari is an experienced firearms examiner and has made an extensive study of the problem of bullet ricochets. Among the papers which he has published on this subject his article, Bullet Ricochet from Metal Plates, appeared in this Journal in September 1969.—EDITOR.

Bullet ricochet is defined as the deflection of a bullet from its course, while maintaining its integrity, as a result of impact on a target of sufficient solidarity. Jauhari (1, 2, 3) was the first to undertake a systematic experimental study of the phenomenon of bullet ricochet by firing low velocity handgun cartridges on targets of such diverse nature as wood, plastics, metal plates, glass, and brick. These experimental studies, apart from providing useful bullet ricochet data, suggested guidelines for building a mathematical model. In the present paper a mathematical model for bullet ricochet has been proposed. The experimental data collected earlier has been interpreted in the light of this model and experiments have been suggested to determine the values of unknown parameters involved in the equations.

MATHEMATICAL MODEL

Let a bullet strike a plane target with velocity V_I and ricochet with a velocity V_R. We assume that the velocity vectors V_R, V_I and the normal to the target at the point of impact lie in one and the same plane which may be called as the plane of fire. Let i and r be the angles of incidence and ricochet as measured from the surface of the target in the plane of fire (see diagram 1).

The incident velocity V_I of the bullet can be resolved into two components; one along the target (V_{IT}) and one perpendicular to it (V_{IN}) in the plane of fire. The following relations follow:

$$V_{IT} = V_I \operatorname{Cos} i, \tag{1}$$

$$V_{IN} = V_I \operatorname{Sin} i. \tag{2}$$

Similarly the velocity V_R of the bullet after ricochet can be resolved into components V_{RT} and V_{RN} given by the relations:

$$V_{RT} = V_R \operatorname{Cos} r, \tag{3}$$

$$V_{RN} = V_R \operatorname{Sin} r. \tag{4}$$

At this stage we introduce two dimensionless parameters α and β defined by

$$\alpha = \frac{V_{RT}}{V_{IT}} = \frac{V_R \operatorname{Cos} r}{V_I \operatorname{Cos} i}, \tag{5}$$

$$\beta = \frac{V_{RN}}{V_{IN}} = \frac{V_R \operatorname{Sin} r}{V_I \operatorname{Sin} i} \tag{6}$$

i.e. they are the ratios of the moduli of component velocity vectors after and before ricochet.

Condition for ricochet: From equations (5) and (6) it can be seen that the condition for ricochet is

$$\alpha \geq 0, \qquad \beta \geq 0 \tag{7}$$

Relations between the angle of incidence and ricochet: Dividing (5) by (6), we get

$$\operatorname{Tan} i = \alpha/\beta \operatorname{Tan} r. \tag{8}$$

Equation (8) gives the dependence of the angle of ricochet on the angle of incidence in terms of parameters α and β. It can be seen from (8) that

$$i \gtreqless r$$

according as

$$\alpha \gtreqless \beta. \tag{9}$$

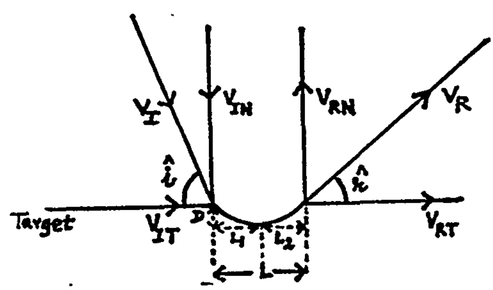

DIAGRAM 1

* Read on the 10th March, 1969 during Winter School on Forensic Science held under the auspices of National Institute of Sciences, India, at Nangal, Punjab.

Thus depending on the values of parameters α and β the angle of ricochet can be less, equal, or greater than the angle of incidence.

Velocity and energy after ricochet: Using equations (5) and (6) and eliminating the terms involving r, we get

$$V_R = V_I\,[\alpha^2 \operatorname{Cos}^2 i + \beta^2 \operatorname{Sin}^2 i]^{1/2}. \quad (10)$$

If W is the mass of the bullet and E_R its kinetic energy after ricochet, we have

$$E_R = \tfrac{1}{2} WV_R^2 = \tfrac{1}{2} WV_I^2[\alpha^2 \operatorname{Cos}^2 i + \beta^2 \operatorname{Sin}^2 i]$$

$$= E_I\,[\alpha^2 \operatorname{Cos}^2 i + \beta^2 \operatorname{Sin}^2 i], \quad (11)$$

where $E_1 = \tfrac{1}{2} WV_I^2$ = initial kinetic energy of the bullet. If E_L is the loss of energy due to ricochet,

$$E_L = E_I - E_R$$

$$= E_I[1 - \alpha^2 \operatorname{Cos}^2 i - \beta^2 \operatorname{Sin}^2 i]. \quad (12)$$

Impulsive force during ricochet: The impact of a bullet on a target is in the form of an impulse, i.e. a large force acting for a very short time. The value of the average force acting on the bullet can be derived by calculating the rate of change of momentum undergone by it during ricochet. If F_T and F_N are the moduli of the components of this force acting along and perpendicular to the target, then

$$F_T = \frac{WV_{IT} - WV_{RT}}{t} \quad (13)$$

and

$$F_N = \frac{WV_{IN} + WV_{RN}}{t}, \quad (14)$$

where t is the time for which the impact lasts. A positive sign has been used before WV_{RN} in (14) because V_{RN} is directed opposite to V_{IN}. Substituting the values of V_{IT}, V_{RT}, V_{IN} & V_{RN} from equations (1), (5), (2) and (6) respectively, we get

$$F_T = \frac{WV_I(1 - \alpha) \operatorname{Cos} i}{t} \quad (15)$$

and

$$F_N = \frac{WV_I(1 + \beta) \operatorname{Sin} i}{t}. \quad (16)$$

The decceleration corresponding to F_T and F_N will be

$$f_T = \frac{F_T}{W} = \frac{V_I(1 - \alpha) \operatorname{Cos} i}{t} \quad (17)$$

and

$$f_N = \frac{F_N}{W} = \frac{V_I(1 + \beta) \operatorname{Sin} i}{t}. \quad (18)$$

If L is the length of the ricochet mark along the line into which the plane of fire and the target plane intersect, we have

$$V_{RT}^2 = V_{IT}^2 - 2f_T \cdot L \cdot$$

Substituting the values of V_{RT}, V_{IT} and f_T from equations (5), (1) and (17) respectively, we get

$$t = \frac{2L}{V_I(1 + \alpha) \operatorname{Cos} i}. \quad (19)$$

Substituting the value of t in equations (15), (16), (17) and (18) we get,

$$F_T = \frac{WV_I^2(1 - \alpha^2) \operatorname{Cos}^2 i}{2L}, \quad (20)$$

$$F_N = \frac{WV_I^2(1 + \beta)(1 + \alpha) \operatorname{Sin}_i \operatorname{Cos} i}{2L}, \quad (21)$$

$$f_T = \frac{F_T}{W} = \frac{V_I^2(1 - \alpha^2) \operatorname{Cos}^2 i}{2L}, \quad (22)$$

and

$$F_N = \frac{F_N}{W} = \frac{V_I^2(1 + \beta)(1 + \alpha) \operatorname{Sin} i \operatorname{Cos} i}{2L} \quad (23)$$

Relation between the length and depth of ricochet mark: If D is the maximum depth of the ricochet mark then V_{IN} i.e. the initial component of bullet velocity normal to the target will be reduced to zero at this depth by decceleration f_N given by equation (23). Hence

$$0 = V_{IN}^2 - 2f_N.D.$$

Substituting the values of V_{IN} & f_N from equations (2) and (23) respectively, we get

$$D = \frac{L \operatorname{Tan} i}{(1 + \alpha)(1 + \beta)}. \quad (24)$$

Equation (24) gives the relationship between the depth and length of a ricochet mark.

If t_1 is the time during which V_{IN} is reduced to zero and the maximum depth D is attained, then

$$0 = V_{IN} - f_N\, t_1.$$

Substituting the values of V_{IN} and f_N from equations (2) and (23) respectively we get,

$$t_1 = \frac{2L}{(1 + \beta)(1 + \alpha)V_I \operatorname{Cos} i}. \quad (25)$$

If L_1 is the distance along the line of intersection of the plane of fire and the target plane from the starting end of the ricochet mark to the point where maximum depth of ricochet mark is attained, then

$$L_1 = V_{IT}\, t_1 - \tfrac{1}{2} f_T\, t_1^2.$$

Substituting the values of V_{IT}, t_1 and f_T from equations (1), (25) and (22) respectively, we get

$$L_1 = \frac{L[\alpha + 2\beta + 1]}{(1+\alpha)(1+\beta)^2}. \qquad (26)$$

If L_2 is the corresponding distance from the terminating end of the ricochet mark, then

$$L_2 = L - L_1 = \frac{\beta^2(1+\alpha+2\alpha/\beta)L}{(1+\alpha)(1+\beta)^2}. \qquad (27)$$

It can be seen from (26) and (27) that the position of maximum depth is asymmetrically placed with respect to the two ends of the ricochet mark. It is also clear that so long α and β are less than unity

$$L_1 > L_2,$$

which means that the position of maximum depth will be nearer to the terminating end of the ricochet mark as compared to the starting end. At the same time it can be visualized that, due to the symmetry of the problem, the ricochet mark will be symmetrical about the line into which the plane of fire intersects the target plane.

Experimental Determination of α, β

The mathematical treatment given above is based on the introduction of two dimensionless parameters α and β defined by equations (5) and (6). It is possible to study their dependence on various factors experimentally and determine their experimental values. In earlier papers (1, 2, 3) the author had devised a simple experimental set up to study bullet ricochet. It was possible by this set up to measure the angle of ricochet at various angles of incidence with sufficient degree of accuracy. By adopting a similar set up the velocity of a bullet after ricochet can also be measured by a counter chronograph provided with photoelectric screens. Knowing the incident velocity, angle of incidence and ricochet the parameters α and β can be calculated by the formulae (5) and (6). The velocity after ricochet will have to be determined in close proximity of the target using minimum possible separation of screens. This is so because a bullet is not only deformed but becomes unstable after ricochet and is therefore likely to follow an erratic path and lose velocity at a rapid rate. In close proximity of the target, i.e. before the bullet has traversed any considerable length, it will be travelling almost in the true direction of ricochet. These considerations had earlier prompted the author to record the point of impact of the bullet after ricochet on a deal board placed very near the target and calculation of the angle of ricochet was done with reference to this point. As the velocity measurements will have to be done with minimum separation of screens, a counter chronograph capable of measuring short time intervals will be ideally suited. It has not been possible to undertake experiments of this nature due to lack of chronographing facilities in the author's laboratory. It is, however, proposed to undertake this work in future when the required facilities are available.

Another practical method by which the values of parameters α and β can be calculated is with the help of equations (8) and (24). As i, r, D and L are measurable quantities, equations (8) and (24) can be solved simultaneously for α and β in terms of i, r, L and D. Solving equations (8) and (24) for α and β, we get

$$\alpha = -\frac{\operatorname{Tan} i}{2\operatorname{Tan} r} - \frac{1}{2} \pm \frac{\operatorname{Tan} i}{2\operatorname{Tan} r}\sqrt{\left(1 - \frac{\operatorname{Tan} r}{\operatorname{Tan} i}\right)^2 + \frac{4L\operatorname{Tan} r}{D}} \qquad (28)$$

and

$$\beta = -\frac{\operatorname{Tan} r}{2\operatorname{Tan} i} - \frac{1}{2} \pm \frac{1}{2}\sqrt{\left(1 - \frac{\operatorname{Tan} r}{\operatorname{Tan} i}\right)^2 + \frac{4L}{D}\operatorname{Tan} r} \qquad (29)$$

Only the positive values of α and β are of interest to us as only then the bullet ricochets.

Thus if i, r, L and D are known the values of α and β can be calculated. Once the values of α and β are known, the values of V_R, E_R, E_L, t etc. are all automatically known if V_I is known in advance. The value of V_I is generally known from the data given by cartridge manufacturers.

A reference to equation (7) shows that it is possible to determine the ratio α/β if only i and r are known. As mentioned earlier this data can be obtained by the method used by the author for bullet ricochet studies. It is generally not possible to obtain the values of r for a large number of values of i at suitable intervals because a bullet fails to ricochet not only by penetrating the target but also by disintegration. Sometime both the conditions prevail simultaneously.

Discussion

The mathematical model proposed above is based on the introduction of two dimensionless parameters α and β which can be determined experimentally either by resorting to velocity measurements or by making measurements on the ricochet mark left by a bullet on a target. The ratio α/β can, however, be calculated if only i and corresponding r are known. To show the order of magnitudes involved, the ratio α/β has been calculated with the help of i, r data obtained earlier in respect of metal plates (2). The calculations are being given in respect of 1/16" & 1/8" thick aluminium plates fired upon by 178 grain .380, ball, MK2, K.F. revolver jacketted bullet at an incident velocity of 600'/Sec.

Target	i in degrees	r in degrees	$\frac{\alpha}{\beta} = \frac{\tan i}{\tan r}$	Remarks
Aluminium 1/16"	15	5	3.1	
	20	10	2.1	
	25	—	—	Bullet disintegrates
Aluminium 1/8"	15	4	3.8	
	30	7	4.7	
	35	8	5.0	
	40	—	—	Bullet disintegrates

It can be seen from above that α/β does not remain constant at different angles of incidence. This implies that either α or β or both vary with the angle of incidence. On the basis of physical reasoning both α and β are expected to vary. Examination of ricochet marks on metal plates showed that these were in the form of dents with permanent deformation of the target at the point of impact. In case of wooden targets the marks were sometime in the form of dents and sometimes showed the evidence of bullet penetrating the target material to some depth. In cases where the marks are in the form of dents without actual penetration of the target material the component of bullet velocity along the target (V_{IT}) is likely to be reduced by forces predominantly frictional in nature. If the forces are frictional in nature the variation of α with the angle of incidence is obvious; since as this angle changes the component of bullet velocity along the target also changes. At high velocities such as are encountered in bullet ricochet the frictional force is likely to vary with velocity. It will also be not independent of the area of contact between the bullet and the target due to the phenomenon of 'seizing' in which there is mutual transfer of material between the two. If a bullet while ricocheting penetrates the target material to some depth the forces retarding the component of velocity along the target are again expected to be velocity dependent. In either case variation in the value of α at different angles of incidence is expected. The parameter β may be identified as the coefficient of restitution. As the impact of a bullet on a target leads to deformation of both and as the deformation is velocity dependent, β is expected to vary with the angle of incidence too. One can visualize that for a stationary target there is nothing to increase the two components of bullet velocity along and normal to the target (V_{IT} and V_{IN}). In fact both will be decreased. Thus both α and β are expected to be less than unity. Further, for ricochet $\alpha \geq 0$ and $\beta \geq 0$; hence for ricochet

$$0 \leq \alpha < 1$$

and

$$0 \leq \beta < 1.$$

This implies that

$$V_R < V_I$$

i.e. the velocity after ricochet will be less than the incident velocity. Although no velocity measurements were undertaken during the course of experiments, the velocity after ricochet appeared to be less than the incident velocity as judged from bullet penetration in 1" thick deal board after ricochet. A bullet normally capable of penetrating several boards was not able to penetrate even one board after ricochet.

In the experiments conducted on wood, plastics, metal plates, glass, and brick (1, 2, 3) the angle of ricochet was always found to be smaller than the angle of incidence. In our model this situation exists for $\alpha > \beta$. The nearer α approaches β the more are the conditions conducive for the equality of angles of incidence and ricochet. Experiments have indicated that the loss of angle on ricochet when the same bullet is fired at the same angle of incidence on different targets depends on the nature of target. For example, in case of 1/2" thick teak wood at 10°

incidence when fired upon by 40 grain .22, Long Rifle, K.F. lead bullet the loss of angle was only 2°. When the same experiment was repeated under similar conditions on brick the loss was as much as 8°. This shows that the ratio α/β was much higher in the latter case as compared to the former.

It has been mentioned earlier that the values of α and β can be determined if i, r, L and D are known. The length of a ricochet mark can be determined quite accurately. Some difficulty may be experienced in determining the depth if the ricochet mark is too shallow or if the target gives way at the point of impact thereby creating a hole without the passage of bullet (1, 2, 3). In an actual crime situation the value of i and r can be approximated by knowing the position of the shooter and the victim with respect to the ricochet mark. L and D can be determined by performing measurements on the ricochet mark itself. The incident velocity can be ascertained from the data provided by the manufacturer of the cartridge used in the commission of crime. All this data can be utilized to compute bullet velocity and energy after ricochet. This calculation may be of great help in assessing the wounding power of a bullet after ricochet and thus help in confirming or refuting a particular theory propounded to reconstruct the shooting incident. An example from the experiments conducted by the author (2) is being given here to show how the calculations are to be performed. During bullet ricochet experiment it was found that the 230 grain, .45 ACP, Rem-Umc bullet ricocheted at an angle of 12°(r) when fired on a ⅛″ thick aluminium plate at 25°(i) incidence with an incident velocity of 800 ft./sec.(V_I). Measurements on the ricochet mark showed that its length was 6.5 cms.(L) and maximum depth 1.2 cms. (D). Using relations (28) and (29), we find that

$$\alpha = .85, \qquad \beta = .38$$

Substituting the values of V_I, α, β and i in equations (10), (11), (12), (15), (16) and (19), we get V_R = 624 ft./sec.; E_R = 6397 foot poundals; E_L = 4117 foot poundals; F_T = 11355 poundals; F_N = 47839 poundals and t = .003 secs. Thus in this case the impact of the bullet lasted for .003 secs. and the bullet lost 4117 foot poundals of energy during ricochet. The loss of energy by the bullet coupled with the fact that the bullet becomes unstable and its ballistic shape is deformed clearly shows that the lethal range of this bullet is likely to decrease considerably after ricochet. Further, if we substitute α = .85 and β = .38 in equation (26) we find that

$$L_I = 4.8 \text{ cms.}$$

The value of L_I as measured on the ricochet mark was found to be 4.6 cms. which is fairly close to that calculated on the basis of mathematical model developed in the preceding paragraphs. In this connection it may be mentioned that if we substitute the values of α and β from equations (28) and (29) in equation (26), we get a relationship between the angle of incidence (i) and the angle of ricochet (r). This relationship also involves L, L_I and D. Thus if i, L, L_I and D are known, one can calculate the value of r which will indicate approximately the path of the bullet immediately after ricochet. This determination can also be of practical interest in the field of investigation.

It can thus be seen that all important features of bullet ricochet observed experimentally (1, 2, 3) are explainable within the framework of this simple mathematical model. It is admitted that the model is rather over simplified in as much as the whole mathematical treatment is based on particle dynamics. A bullet is not a mere particle but a body having finite dimensions. It is also spinning at the time of impact. It is, therefore, necessary to examine how far the quantitative results obtained on the basis of this model differ from those observed experimentally. It would be interesting to determine the velocity after ricochet experimentally by a chronograph and see how much it differs from that calculated by performing measurements on the ricochet marks. If the deviation is excessive the model will have to be suitably modified for practical work. It has been seen earlier that the value of L_I as calculated on the basis of this simplified model in a particular case does not differ much from that obtained experimentally. It is felt that all such calculations will have to be performed in a large number of control firings and the results compared with the experimental values. Work on these lines is in progress.

REFERENCES

1. Jauhari, M., Ricochet of Cartridge, S.A., Ball, Revolver, .380, MK2, K.F. Bullet, Jour. Ind. Acad. For. Sc., Vol. 5, No. 1, pp. 29–33, 1966.
2. Jauhari, M., Bullet Ricochet from Metal Plates, Jour. Crim. Law Crimin. & Police Sc., Vol. 60, No. 3, pp. 387–394, 1969.
3. Jauhari, M., On the Ricochet of .22, Long Rifle, K.F. Bullet. Jour. Ind. Acad. For. Sc. Vol. 9, No. 1, pp. 14–18, 1970.