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OPTIMAL SELECTION OF POLICE PATROL BEATS

PHILLIP S. MITCHELL

Dr. Phillip S. Mitchell is a Law Enforcement Consultant and an Associate Professor of Quantitative Methods at California State University, Fullerton, California. He is active in consulting, research, and teaching in the areas of mathematical programming and public systems modeling. Dr. Mitchell is an associate member of the International Association of Chiefs of Police and serves on the Research & Development Task Force of the California Council on Criminal Justice.

There has been a notably increased pressure on law enforcement agencies across the nation to use their manpower more efficiently. The major contributing factor appears to be the increasing per capita crime rate without corresponding increases in law enforcement resources. This pressure has led to an interest on the part of chief officers and other decision makers in those techniques of operations research which can be used to provide better service through the efficient allocation and distribution of manpower.

The distribution of manpower over patrol beats has been accomplished historically on an empirical basis using hand calculations. The primary criterion used in determining beat structure has been the equalization of work load or, as a surrogate, the equalization of the percentage of incidents occurring within the beat boundaries. It has been impossible to arrange the geographic distribution of beats so as to obtain the best possible mean response time, with equal work loads, using hand calculations.

The advent of computer based methods of optimization has made the determination of beat structure using advanced mathematical techniques economically feasible. It is the purpose of this paper to present practical static optimization models for the efficient geographic distribution of police patrol manpower. Although statistically based, the models are analytic in nature and can be solved quite accurately by heuristic methods on a digital computer.

THE BASIC OPTIMIZATION MODEL

We assume that the municipality or the region under study may be partitioned into geographic subunits, with each geographic unit on the order of a one-fourth mile square. Of course the smaller the subunits the more accurate the model but the greater the cost of data collection and handling. We also assume the incident distribution, over both space and time, is known, and that a distance

measure or metric between the centers of each sub-unit is available. Finally, we assume that the nearest available unit responds to a call.

Then our problem is basically one of "clustering" or associating the geographic subregions into larger groups—patrol districts or beats—in such fashion as to maximize or minimize some objective and possibly subject to some constraints. Suppose we now establish the following conventions: let

- A represent the global partition of the region, with the subregions indexed by i ,
- A_k be the k -th order subset of A, i.e., an element of the class consisting of the $n!/(k!(n-k)!)$ possible subsets of A containing k elements, where k is the number of districts into which the region is to be partitioned for beats,
- $d(i, j)$ be the distance or metric from the centroid of i -th subregion to the centroid of the j -th subregion over the best route, and
- $p(j)$ the expected number of calls for service in the j -th subregion over the time period.

Then if we accept the minimization of total weighted travel distance—and hence implicitly the expected travel distance to service a call—as an objective, we may state a simple model fulfilling our requirements as

$$(1) \quad \text{Minimize } \sum_{A_k} p(j) \text{ Minimum}_{i \in A_k} d(i, j).$$

Although considerably less satisfying, an objective function which minimizes the maximum weighted travel distance may also be useful. This takes the form

$$(2) \quad \text{Minimize}_{A_k} \left\{ \text{Maximum}_j \left\{ \text{Minimum}_{i \in A_k} d(i, j) p(j) \right\} \right\}.$$

Although the models above are generally quite useful, the basic expected value model falls short of the state of the art in several respects. First, it considers only the number of calls in each region even though different types of calls have different service time requirements and even though the distribution of calls by type may vary considerably

over the region. In addition, the objective (1) considers only the nearest unit response. The basic model may be broadened to include these considerations.

Given some incident classification scheme, let

- $p(j, m)$ be the expected number of incidents of type m occurring in the j -th subregion over the period and let
- $w(m, q)$ be a subjective weighting factor for the q -th unit responding to an incident of type m .

That is, if a certain type incident requires response of only the two nearest units, $w(m, q) = 0$ for $q > 2$. For the first and second cars responding ($q = 1$ or 2) the value of w represents the relative importance weighting of a rapid response. For incidents considered hazardous to life or patrol preventable, w might be large, with a smaller value for calls which do not require an emergency response. Thus we are in a position to allow decision makers to utilize their own subjective evaluation of the importance of various types of incidents.

Finally, let

- minimum^q be the q -th minimum over the set A_k . That is, minimum^q represents the minimum over the set A_k after each of the $q - 1$ previous minimizing elements have been removed.

Then we may state our objective function as

$$(3) \quad \text{Minimize } \sum_{A_k} \sum_j \sum_m \left\{ p(j, m) \sum_q w(m, q) \text{minimum}^q d(i, j) \right\}$$

where each "subminimum" selects the second, third or more backup units. Thus the objective function of (3) simultaneously accounts for the subjective weighting factors and multiple unit response.

THE WORK LOAD CONSTRAINT

Direct utilization of the basic unconstrained model may result in an unsatisfactory allocation of resources if the incident distribution is not uniform. That is, minimizing the average overall response time will often cause significant differences in beat work loads and response times. Areas of the region in which the incident frequency is low relative to the distances which must be traveled in order to provide service will have relatively low work loads and relatively high response times, with

the converse occurring in the high incident density areas. Although this does not seem unreasonable, in practice the differences are too great to be acceptable to patrol commanders. It is therefore of considerable importance that the beats be defined with a requirement of equal or nearly equal work loads.

In constraining the workload we need the following definition. Let

- $s(m)$ be the typical service time requirement for each of the types of incident in the classification scheme.

Then the average incident load for the k' -th beat over the period, disregarding the fact that a small percentage of each beat's work load is generated by backup calls, is defined by

$$(4) \quad S(k') = \sum_m \sum_{j \in R(k')} s(m)p(j, m)$$

where $R(k')$ is the set of subregions making up the k' -th beat. An acceptable definition of work load should include response time as well as service time. If we let $t(k')$ represent the average driving time required to service an incident in the k' -th beat, we may define work load as

$$(5) \quad L(k') = S(k') \sum_m \sum_{j \in R(k')} t(k')p(j, m)$$

The work load constraint simply amounts to the requirement that $L(k')$ be equalized for all of the k' beats.

OTHER CONSTRAINTS

Most of the criticisms of the simple expected value model of (1) above may be satisfied without going to the min-max model of (2). One method is to use the square of the distance in the objective function, thus tending to weight the greater distances more heavily. Also, a distance constraint of the form

$$(6) \quad \text{Maximum}_j \left\{ \text{Minimum}_{i \in A_k} d(i, j) \right\} \leq T(j) \text{ for all } j$$

may be added, where $T(j)$ is a constant for each of the subregions. In practice, this constraint may be handled quite satisfactorily through the artifice of a penalty function reformulation. Suppose we define

$$(7) \quad G(A_k) = \begin{cases} 0 & \text{if the constraint is satisfied and} \\ M, M >> 0 & \text{if unsatisfied.} \end{cases}$$

The expression $G(A_k)$ may be added to the objective function (3) to provide the appropriate result.

HEURISTIC COMPUTATIONAL ALGORITHMS

Allocation problems of the type stated in (1) above may be simply restated, for expository purposes, as

$$(8) \quad \text{Minimize } \sum_{A_k} \sum_j \text{minimum } W(i, j).$$

where $W(i, j)$ is the matrix of appropriately weighted distances. Additional considerations needlessly complicate the presentation and are dropped for the present.

The structure of the problem stated in expression (8) above is actually quite simple. The assumptions of the model require that $W(i, j)$ be a distance matrix with the (i, j) -th element representing the weighted travel or other distance from the i -th location to the j -th location. The objective is then to choose a subset of k rows of W in such fashion as to minimize the sum of the column minimums, where each column minimum is chosen only from among the designated subset of k rows.

The problem statement is quite straightforward and solution by enumeration is easy for small problems. However, as the problem matrix is allowed to reach interesting proportions solution by enumeration becomes impossible. Hence, the primary barrier to enumeration in such problems is not computer memory limitation, since a 200×200 matrix requires only 40,000 words, but sheer computational expense. Heuristic algorithms offer an alternative method of "solution" which is quite economic in most applications. They will be discussed only briefly here, since current methods were summarized by ReVelle, Marks and Leibman in their recent article¹.

Heuristic algorithms of the type generally proposed for allocation or clustering problems often have two phases. In the first phase, k locations are selected in some fashion. The second, or improvement phase, then seeks to improve on locations selected in the first phase, perhaps by sequential substitution of the locations selected. The first phase selection may be done in several ways. One method selects initially one location, and then keeps adding more locations to the allocation while minimizing the objective function at each step until the allocation reaches k . A second approach begins with the whole feasible set as an initial allocation and sequentially reduces the set

by eliminating the "worst" location until finally only k locations remain.

An improvement routine due to Teitz and Bart² operates as follows. A location not in the (current) allocation is successively substituted for each of the current members and the value of the objective function calculated. If the best value of the objective function is not superior to the original, the original is retained. Otherwise, a substitution of the location under test for the location (in the current allocation) showing the most improvement in the objective function is made. The process is repeated for each of the locations not in the allocation until no improvement is made after a complete cycle.

Maranzana³ begins with an arbitrary selection of k locations and partitions the region in such fashion that each of the subregions is served by the nearest of the k locations. For each of the k clusters or groups thus formed, the local center of gravity is determined. In those cases in which the local center of gravity is different from the originally chosen location, the center of gravity is substituted for the originally chosen location. The algorithm terminates when no further changes can be made. Several (random?) initial selections may be made and the results compared.

APPLICATION

Preliminary tests of the basic model of equation (1) above have been successfully carried out using one year's incident data for Anaheim, California, a rapidly growing southern California city of some 180,000 people. The city was broken into 221 subregions corresponding to the quarter section plan upon which the original layout of the city was based. Most of the major traffic arteries lie along the quarter section boundaries, and the streets are generally perpendicular, so that "block distance" appeared to be the most appropriate distance measure. The only complication in the distance calculation was caused by the Santa Ana Freeway, which cuts the city diagonally into two parts. This freeway has approximately six under- or overcrossings within the city limits, so that the distance between two points on opposite sides of the freeway had to be calculated accordingly.

Figure 1 illustrates the overall percentage of

² Teitz & Bart, *Heuristic Methods for Estimating the Generalized Vertex Median of a Weighted Graph*, 16 OPERATIONAL RESEARCH (1966).

³ Maranzana, *On Location of Supply Points to Minimize Transport Costs*, 15 OPERATIONAL RESEARCH QUARTERLY (1964).

¹ ReVelle, Marks & Leibman, *An Analysis of Private and Public Sector Location Models*, 16 MANAGEMENT SCIENCE, 11 (1970).

The results of this simple deterministic simulation showed only negligible differences in the overall mean response distance. Distances were very slightly higher, as would be expected, for the ten, eleven, and twelve beat plans, but differences were negligible thereafter. This is not a particularly startling result, since the majority of all incidents do not require multiple unit response and are serviced by the unit which belongs on a particular beat. A more complex and definitive stochastic simulation test which will examine more of the systems behavior is now being undertaken.

From table 1 an interesting result is evident at a glance. While the addition of each new patrol unit to the actual beat plans did decrease the average response distance, the amount of the decrease diminished as the number of beats increased. The heuristically developed beats showed the same tendency, but to a much less marked degree. It seems clear that while the human mind is soon unable to comprehend the effects of individual changes on the whole plan, the computer has no such failing. If these same tendencies hold for even larger regions, it is easily seen that the computer is capable of making very significant differences at the thirty or forty beat level.

Since the primary objective of patrol is response to called for services, especially those of an urgent nature, a good case can be made for a direct relationship between satisfaction of this objective and diminution of mean response time. It is always difficult to impute a more general meaning to a simple measure such as average response time. However, the transition, though dangerous, is worth the effort. With this in mind table 1 may be used to give some feeling for the value of added patrol units.

For example, notice that the hand developed beat plan had an overall mean response distance of 1.51 units at the 15 beat level, while the computer developed beats had an overall mean response distance of 1.49 units at the 12 beat level. Similarly, the mean response distance for the constrained optimal 15 beat plan is 1.24, for a decrease of about 18 percent. Decision makers then have the option of either holding the capital cost of patrol at the level indicated by 15 units and minimizing response distance, or of maintaining current response distance and lowering the number of units and therefore the cost of patrol. Combinations of both are, of course, possible. The latter of these two alternatives, the reduction of patrol

TABLE 1
COMPARISON OF BEAT PLANS

Beat Plan	Optimization on Distance Only		Optimization with Incident Load Constraint		Actual Beats	
	Mean Travel Distance	Incident Range (Percent)	Mean Travel Distance	Incident Range (Percent)	Mean Travel Distance	Incident Range (Percent)
10	1.52	12.7	1.73	3.2	*	
11	1.41	12.7	1.59	3.5	*	
12	1.34	9.1	1.49	2.9	*	
13	1.27	8.4	1.39	4.3	*	
14	1.21	6.5	1.34	2.8	1.55	6.0
15	1.17	6.4	1.24	4.2	1.51	6.3
16	1.11	6.1	1.22	5.0	1.46	6.0
17	1.07	6.0	1.15	3.0	1.45	5.6
18	1.03	6.0	1.09	4.8	1.37	4.1
19	1.00	5.9	1.04	4.1	1.30	6.1
20	0.97	5.1	0.98	4.5	1.29	5.9
21	0.94	4.5	0.94	4.5	*	

* Unavailable.

units, is generally not feasible. However, the optimization of expected response distance has an evident value of its own, and might aid in holding the budget line on patrol so that resources could gradually be shifted to other areas such as detective or narcotics bureaus.

COMPARING BEAT PLANS

It is instructive to observe the differences in beat plans for at least one case. Figure 2 shows the 14 beat plan developed by hand while Figure 3 shows the heuristically developed plan. It would appear that the freeway was uppermost in the mind of the designer, for the beats seem to be developed around this natural obstacle which may be seen running from the upper left to lower right hand corner of the map. Comparatively, the computer developed beat plan used the freeway as a boundary only in the central section of the city, while beats three and twelve may be seen to encompass the freeway itself. This tendency on the part of the human designer to work around the freeway held for all the best plans. Every existing beat plan was developed using the freeway as a boundary. This result might well indicate that it is too difficult for a human decision maker to take the freeway into account in his "eyeball" distance calculation, a conclusion which certainly does not contradict common sense. If it can be generalized, this conclusion might lead us to believe that it is even more

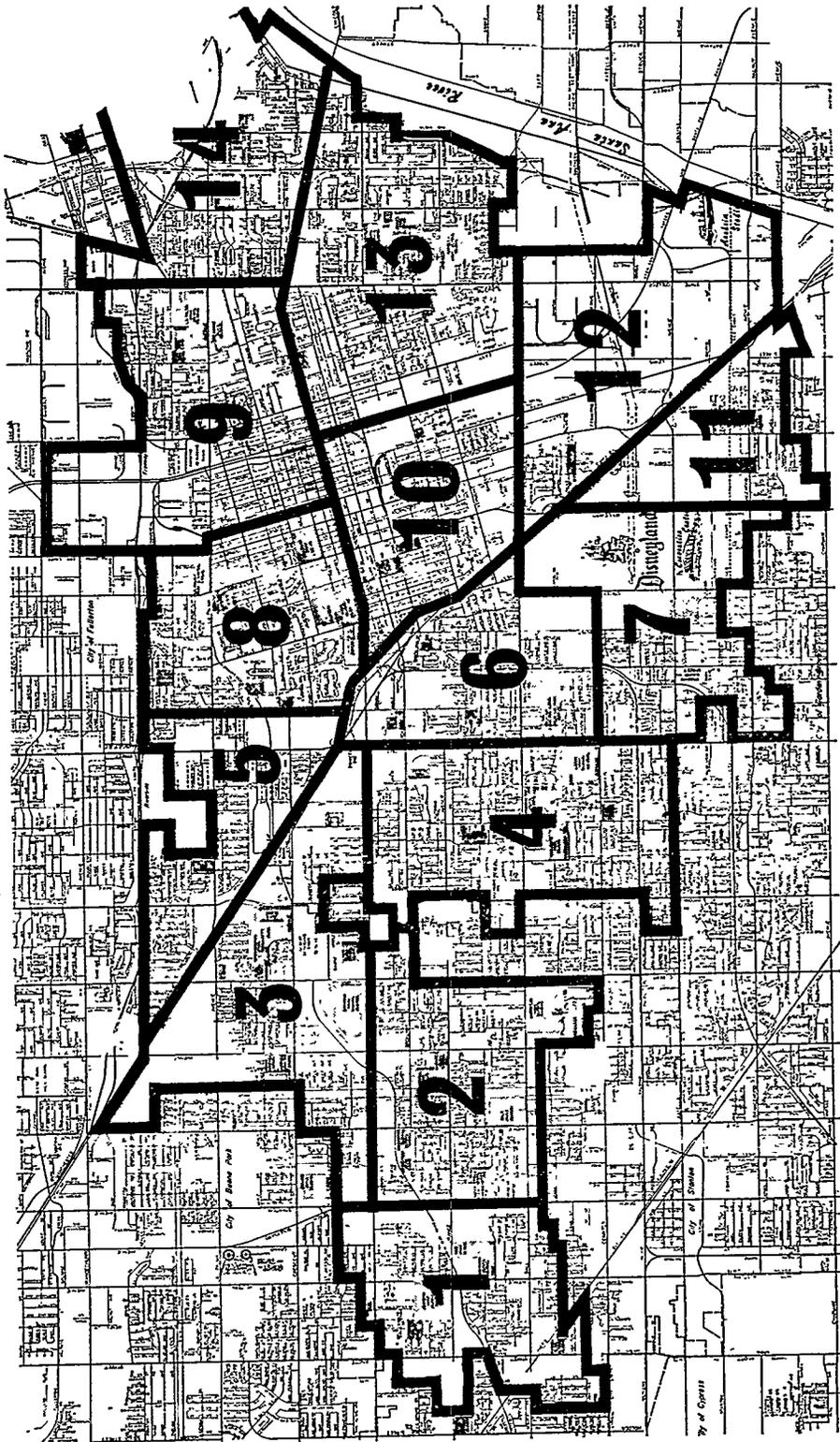


FIGURE 2.

Actual beat plan.

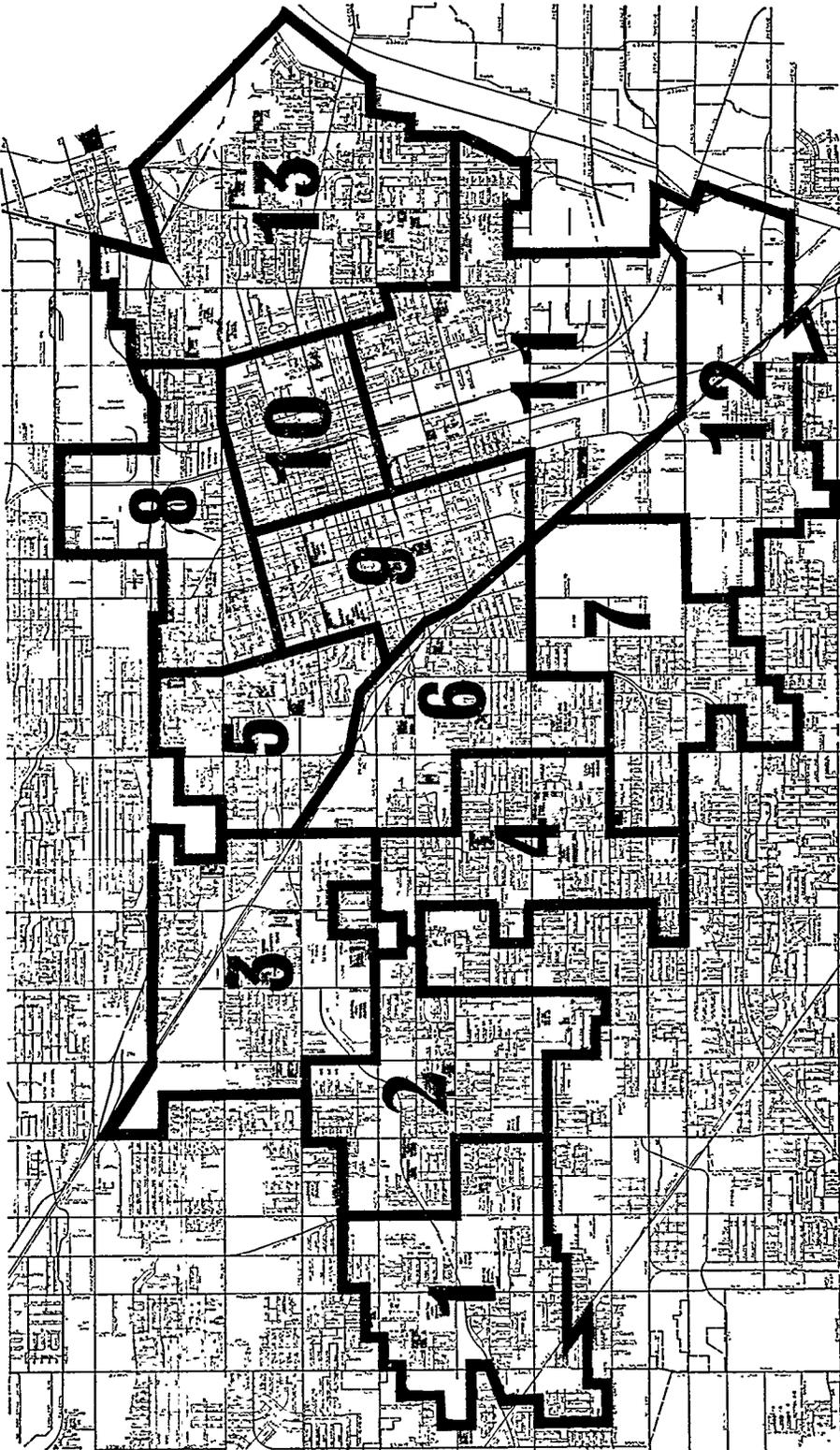


FIGURE 3.

Computer developed beat plan.

difficult to do mental juggling with several natural boundaries in a larger jurisdiction, so that the computer-based solutions might be even more useful in these larger jurisdictions. A test of this conclusion in a larger region is now being proposed.

CONCLUSION

The primary advantage of patrol districting through minimization of expected weighted response distance is obvious. A second advantage is

to increase the time available for preventive patrols. Also, clustering by minimization of travel distance automatically increases patrol frequency in areas having a high incident rate simply by the fact that beats tend to have nearly equal incident loads even without explicit use of a constraint, so that high incident districts have beats which are smaller geographically. Thus the two primary functions of patrol, answering calls for service and deterrence, are simultaneously satisfied.