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APPROXIMATE RELATIONSHIP BETWEEN THE ANGLES OF INCIDENCE AND RICOCHET FOR PRACTICAL APPLICATION IN THE FIELD OF CRIMINAL INVESTIGATION

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In earlier papers (1, 2, 3, 4) the author had reported the results of bullet ricochet experiments carried out on targets of diverse nature by firing low velocity handgun cartridges. These experiments brought out a very significant qualitative relationship between the angles of incidence and ricochet viz. the angle of ricochet is less than the angle of incidence. For the proper reconstruction of a shooting incident it is, at times, desirable to know the path of the bullet before and after ricochet. This, *inter alia*, requires a knowledge of the relationship between the angles of incidence and ricochet. If a mathematical relationship between the angles of incidence and ricochet is known, it will be possible to predict the one provided the other is known. Apart from the qualitative relationship mentioned above no mathematical formula connecting the angles of incidence and ricochet exists in literature. In the present paper a simplified treatment based on two dimensional particle dynamics has been given to derive such a relationship in terms of the dimensions of a ricochet mark. This has resulted into a very simple formula connecting the angles of incidence and ricochet which may be useful for practical application in the field of criminal investigation.

Let TT′ be a plane target and a bullet strike it at 0 with an incident velocity V_I at an incidence angle i as measured from the target (see Diagram 1). As a result of impact the bullet dents/penetrates the target thereby producing a ricochet mark whose section in the plane of incidence/ricochet (assumed to be the same) is shown as OAB. The bullet then loses contact with the target at B and ricochets with a velocity V_R at a ricochet angle r as measured from the target.

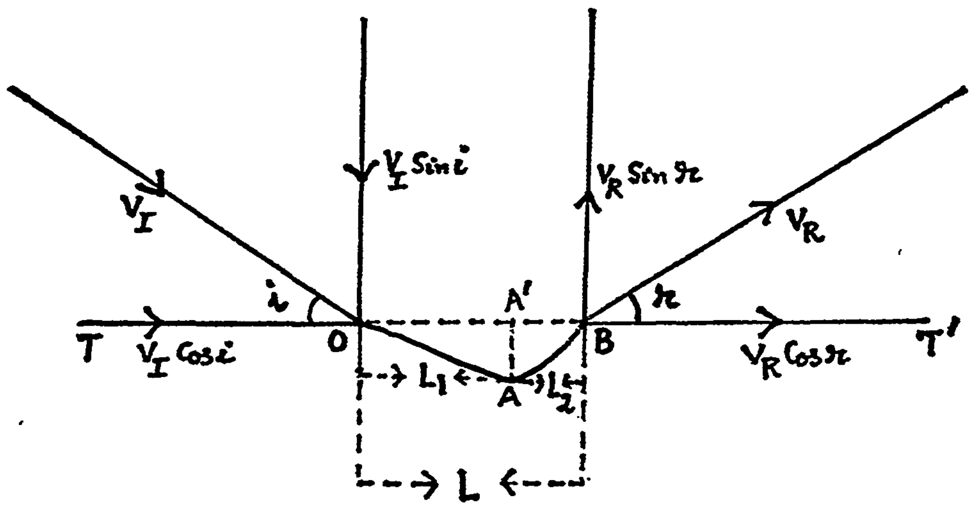

DIAGRAM 1

The initial velocity V_I of the bullet can be resolved into two components; one along the target (V_I Cos i) and the other perpendicular to the target (V_I Sin i). The component of bullet velocity along the target (V_I Cos i) is reduced during the time of impact by forces predominantly frictional in nature. As regards the component of bullet velocity perpendicular to the target, we may consider the impact as consisting of two stages. During the first stage of impact the component of bullet velocity perpendicular to the target is reduced from its initial value (V_I Sin i) to zero. When this component becomes zero there is no further denting/penetration of the target and hence the maximum depth in the ricochet mark is attained at A. Further, both the bullet and the target undergo deformation during this period and the kinetic energy amounting to $\frac{1}{2} WV_I^2 \sin^2 i$ (W = mass of the bullet) gets stored as the internal potential energy of deformation. During the second stage the elastic forces of the bullet and the target come into play and the stored internal potential energy tries to convert itself into the kinetic energy of motion. As neither the bullet nor the target are perfectly elastic bodies, both of them suffer permanent deformation and hence only a part of the stored internal potential energy is able to convert itself into the kinetic energy of motion. The rest of the stored energy is lost. For a stationary target, therefore, the bullet ricochets with a reduced velocity.

The time of impact is extremely short and during this short time interval the linear momentum of the bullet undergoes a finite change. The forces acting on the bullet during ricochet are impulsive in nature. It is very difficult to guess the exact mathematical form of these forces as they are extremely complicated and vary within wide limits during a short interval of time. One can affect great simplification in calculations by substituting their time average in place of the actual forces. If we resort to this approximation, the average impulsive force acting perpendicular to the target ($\overline{F_N}$) can be written as

$$\overline{F_N} = W\frac{(V_I \sin i + V_R \sin r)}{T} \qquad (1)$$

where

T = time of impact.

If β is the coefficient of restitution, we have

$$\beta = \frac{V_R \sin r}{V_I \sin i} \qquad (2)$$

Substituting for V_R Sin r from (2) in (1), we get

$$\overline{F_N} = \frac{W(1 + \beta)V_I \sin i}{T} \qquad (3)$$

As an approximation let us neglect the effect of friction. This implies that the component of initial bullet velocity along the target will remain unchanged due to ricochet.
Thus

$$V_R \cos r = V_I \cos i \qquad (4)$$

From (2) & (4), we get

$$\beta = \frac{\tan r}{\tan i} \qquad (5)$$

As β is the coefficient of restitution, we should have $\beta < 1$ or from (5) $r < i$. Experimental observations on bullet ricochet from targets of such diverse nature as wood, plastics, glass, brick, and metal plates show that $r < i$ (1, 2, 3, 4). Thus the assumption of neglecting the effect of friction does not lead to contradiction with actual experimental observations.

Now, let t_1 be the time taken by the bullet to reach the point of maximum depth A. Then

$$t_1 = \frac{L_1}{V_I \cos i} \qquad (6)$$

where

L_1 = distance of the point 0 from the foot of perpendicular from A on the target TT′.

Further, since the component of bullet velocity perpendicular to the target has a zero value at the point of maximum depth A, we have

$$0 = V_I \sin i - \frac{\overline{F_N}}{W} t_1.$$

Substituting for $\overline{F_N}$ from (3), we get

$$t_1 = \frac{T}{1 + \beta} \qquad (7)$$

Now, during the time of impact T, the distance covered along the target by the bullet is OB i.e. the length of the ricochet mark. Hence, if L is the length of the ricochet mark, we have

$$T = \frac{L}{V_I \cos i} \qquad (8)$$

TABLE I

COMPARISON OF EXPERIMENTAL AND CALCULATED VALUES OF i & r

Target	Experimental Value of i to the Nearest Degree	Experimental Value of r to the Nearest Degree	L_1 (in cms.)	L_2 (in cms.)	Calculated Value of i to the Nearest Degree	Calculated Value of r to the Nearest Degree
1/16″ Aluminium*	10	6	3.0	1.6	11	6
1/16″ Aluminium†	10	7	7.5	4.9	10	7
1/8″ Aluminium†	25	12	4.6	1.9	27	11
1/16″ Brass†	15	11	3.7	2.7	15	11
1/16″ Steel†	15	7	4.0	2.5	11	10
1/16″ Steel†	30	10	3.6	1.9	19	17

* Firing conducted with a .22 rifle.

† Firing conducted with a .45 pistol.

Substituting for T from (8) in (7), we get

$$t_1 = \frac{L}{V_1(\cos i)(1+\beta)} \quad (9)$$

Equating the values of t_1 from (6) & (9), we get

$$L_1 = \frac{L}{1+\beta}$$

or

$$\beta = \frac{L - L_1}{L_1} = \frac{L_2}{L_1} \quad (10)$$

where L_2 is the distance of point B from A′ along the target TT′.

Substituting for β from (10) in (5), we get

$$\tan r = (\tan i)\left(\frac{L_2}{L_1}\right) \quad (11)$$

Equation (11) gives a very simple relationship between the angle of incidence and ricochet. If L_2 & L_1 are measurable, r can be determined provided i is known and vice-versa. Unless the ricochet mark is very shallow or the target breaks in the region of impact, it is generally possible to locate the point of maximum depth A with reasonable degree of accuracy. Experiments (5) also indicate that $L_2 < L_1$. Hence β as given by (10) will be less than unity. Thus the value of r as calculated from (11) will always be less than i as observed experimentally. To ascertain how accurately (11) predicts the value of i & r, experimental data in respect of i, r, L_1 & L_2 was, earlier, obtained on plane targets of aluminium, brass and steel (1/16″ & 1/8″ thick). The experimental data obtained in six of the firings where L_1 & L_2 were measurable without ambiguity is given in Table I. The value of i was first calculated with the help of (11) by feeding experimental data for r, L_1 & L_2. Again, the value of r was calculated with the help of (11) by feeding experimental data for i, L_1, L_2. These calculated values are also given in Table I. It is seen that the calculated values of i & r with the help of (11) are in excellent agreement with the observed values of i & r except in case of last two examples where the agreement can at the most be rated as fair.

In criminal investigation the value of i/r can be approximated by knowing the position of the shooter/victim with respect to the ricochet mark. In certain cases the bullet may pass through a curtain or similar article before or after striking the target. The position of the bullet hole on the curtain, etc. can also help in providing an estimate of i/r. Some experimental firing with the suspected firearm/ammunition may be conducted on the target bearing the ricochet mark at known angles of incidence and the value of r, L_1 & L_2 so determined may be fed into (11) to ascertain whether the relationship (11) holds for the particular combination of firearm, ammunition and target involved in crime. The relationship is expected to hold good for those combinations where the effect of friction can be neglected. In other cases it may hold only roughly. After ascertaining the applicability of (11), the data obtained from the ricochet mark caused by the crime bullet and the estimate of i or r obtained as mentioned above may be fed into (11) to obtain r or i as the case may be. This will help in tracing the path of bullet in an approximate manner. The simple formula (11) can, therefore, be of practical interest in the field of criminal investigation.

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