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An Application of Game Theory to Police Problems

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In presenting this paper, the editorial board recognizes a new type of research tool which Mr. Smith has applied to police problems. In his article, the example cited has been reduced to the simplest terms, and to many experienced police officers the ultimate conclusion may seem obvious. The article is not presented with the idea of solving this particular simplified problem, but rather to call attention to the methods that are available and to arouse interest of our readers in the application of game theory to police problems.—Editor.

This article has been developed for two purposes. First it is intended to demonstrate an application of a relatively new operations research technique (Game Theory), and secondly, in the course of the demonstration, to analyze one of the oldest precepts of crime prevention.

Game theory was first introduced about three decades ago by John von Neumann. In 1944 von Neumann and Morgenstern presented a work which applied the theory to certain aspects of economic behavior. Since that time the theory has become increasingly useful, and with the development of electronic computers it has become a very powerful problem-solving technique.

The analysis presented here is a simple one. The reader will be taken step by step through the process of the decision game and in the end, if all goes well, the possibility of other applications in the police field will become apparent.

Game theory, as the name implies, relates to situations in which a game-like conflict exists between two persons or parties. This conflict need not be as harmless as those found in ordinary parlors. It can, and often does, involve life and death. Application of the theory has been made to such problems as, searching for enemy submarines, the deployment of field forces in battle, determining the locations and types of industrial or military target areas to be bombed.

To demonstrate the application of game theory as an analytical tool, we will examine the conflict between the criminal and the police. Admitting that this is a very large and complex conflict, we will, for the purpose of demonstration, limit ourselves to one important part. We will analyze the conflict between criminals in general and a single beat patrol officer.

There are two forces of interest in this conflict:

1. The criminal wants to commit his crime and escape apprehension.
2. The officer wants to prevent the crime and/or apprehend the criminal.

This is clearly a conflict of interest. We are interested, of course, in helping the officer. There are several things we could do to assist the officer. First, we could remove all persons from his beat area and erect a barricade around it. This would certainly prevent a considerable amount of crime in that area. Secondly, we could assign about 1,000 extra officers to him and allow him to use them where he saw fit. Neither of these ideas is practical for obvious reasons. Finally, we can help the officer analyze his problem and perhaps, if our thinking is straight, we may be able to insure him some advantage over the criminal, or at least, failing to give him a definite advantage, we can protect him against the worse possible disadvantage.

Elements of the Problem

What are the elements of the patrolman's problem? There are thousands of them, of course, but let us be concerned only with a few of the more obvious and more important ones. First the officer (call him Waldo) can be in one, and only one, place at a given time. He cannot always make his move (take his turn at play) since he is not always available to move. Waldo drinks coffee and eats lunch, etc.

Waldo, for the past year, has been keeping a score card accounting for each crime that has occurred on each of the four blocks in his beat. He therefore knows what has happened and when and where it has happened. He also knows that it is more important to prevent certain more serious crimes than others.

Waldo already has one slight advantage. Looking at his score card he can tell which blocks are most often attacked during which hours of his tour of duty. He reasons: "If I figure out the percentage of crimes by block, then all I need to do is divide my time according to the same percentage, and spend that amount of time on each block." He reasons further: "If I do that, the criminal will soon figure that I am spending a certain amount of time in each block and he will make it a point to be somewhere else in my beat. What I must do is figure a way to be where I am needed as often as possible, *but I must at the same time be unpredictable.*"

Waldo's problem boils down to this: He must recognize that the criminal will take advantage of him *if he becomes systematic* about his patrol work. At the same time, he cannot figure a way to be where he should be without being systematic. The dilemma has a solution, but it is not necessarily the kind of a solution we find in fairy tales. We cannot guarantee to Waldo that he will always be where he should be. We can guarantee to him, however, that we will maximize the minimum winnings for each of his moves in the conflict game. His "winnings" in this game, of course, are the points he gets for being where he should be.

This idea of maximizing a minimum may seem a bit strange at first. It is very important that it is completely understood, however. To maximize a minimum does not mean that we are going to win as much as possible at each play of the game. It does mean that we are going to keep the least possible amount we can win at each play of the game at its highest possible level.[1]

As small as Waldo's problem is, it is much too complex for him or anyone else to solve unless it is reduced to a form where by it may be manipulated. Game theory does this for him. What we must discover is a set of strategies (moves) that will yield the largest return of the minimum winnings for each of Waldo's plays in the game. We want to tell Waldo how many times he should be in each block in his beat, and also the order in which he should cover the blocks in his beat in his conflict with the criminal. (How many times he should make a particular move and when he should make it.)

In our solution we will be working with what is called a "Game Matrix."[2] We will confine our illustrative problem to a small 2 x 2 (two by two) matrix since they are the easiest to handle.

A Practice Set. Waldo is an accomplished beat officer. He is assigned the task of protecting a bank and a melon patch. At any one time he may protect either, but not both, against attack.

Oswald is a criminal, and he is interested in stealing from either the bank or the melon patch. He may steal from either, but not both, at any one time.

For our purposes the values of the places are the same to both the officer and the criminal. They both figure that the melon patch is worth 1 point while the bank is worth 3 points. If, after attack, both places are still intact, the winning is 4 points to Waldo. If the melon patch is lost then the winning is three (he still has the bank). If the bank is lost the winning is only 1. The points Waldo wins are the losses of Oswald, and vice versa.

		Oswald	
		Column 1	Column 2
Waldo	Row 1	4	melon patch 1
	Row 2	bank 3	4

FIGURE 1

Looking at the game matrix (figure 1), we see that in order for Oswald to sack the bank he must attack Column 1, to sack the melon patch he must attack Column 2. Waldo must likewise defend Row 1 in order to intercept Oswald before he reaches the bank and thus protect it. Similarly, he must

[1] EDITOR'S NOTE: Good poker players use this principle. In stud poker, for instance, on the second draw, a good player will not remain in the game with two deuces when two kings are showing.

[2] A matrix as here used is simply a rectangular array of numbers arranged in rows and columns as follows:

1	2
3	4

defend Row 2 to protect the melon patch. The result of each play is determined by the square in which the opponents meet.

Now, if Oswald attacks Column 1 and by some unfortunate accident Waldo decides to defend the melon patch by going to Row 2, Waldo will lose the bank and with it three points to the scoundrel Oswald. What can Waldo do to give protection to both the bank and the melon patch, and hold his minimum possible winnings to a maximum? He must compute the game matrix and make his move accordingly.

There are several techniques available for computing the game matrix in this problem. Regardless of which one we use, the results will be the same. Since we are concerned with demonstrating a general problem solving technique it would seem appropriate to use the simplest computational method available. It is not necessary to understand the properties of determinants to understand the value of the solution to the game matrix. We are primarily concerned with the what and why of the solution and only academically interested in the technical part of the problem.

The game matrix in figure 1 represents our best estimate of the problem faced by the Patrolman Waldo. He has available two alternative moves, each of which carry some measure of reward. Both available moves are demanding in the sense that he is responsible for the protection of both property locations, the bank and the melon patch. Obviously, the bank is of greater importance which is why we assigned a greater value to it (3 points as opposed to one for the melon patch).

Taking the problem back to the real world we find our officer standing somewhere between the bank and the melon patch, and he is trying to decide to which of the two he should go at this particular moment. He is faced with the uncertainty, indeed, he is completely ignorant, of which of the two places will be attacked in the next moment.

Our officer has a righteous feeling that tells him he should spend more time protecting the bank since the loss to society in general could be greater in the event of attack there than it would were a few melons lost. Still, the rules demand that he protect both from the criminal. Common sense tells the officer that if he spends all of his time protecting the bank the criminal will soon become aware of this and spend all of his time stealing melons. On the other hand, if the officer sits in the melon patch the criminal will soon empty the bank of its treasures. In either event the criminal would be at a constant advantage and the officer at a disadvantage.

Waldo must devise a scheme which will provide him with a set of strategies or moves such that both the bank and the melon patch are protected, and in the game matrix he wins as many points as he can within the rules of the game. Since he is never sure just what the criminal is going to do, he wants to protect his bank and the melon patch against anything the criminal *might* do. (If he had sure knowledge of the criminals movements he would have few problems). In order that Waldo may insure himself that the least he can win at any play of game will be at a maximum, he must assume one thing. He must consider that the criminal is an intelligent and clever opponent who is not going to do anything stupid such as set up a predictable pattern of moves or attack one or the other place all of the time neglecting the other. Waldo must assume that the criminal will be at least as clever as himself.

Operating on this assumption will protect Waldo from the cleverest criminal and, of course, if the criminal does anything stupid Waldo will gain from it. Unfortunately, as we will learn later, the reverse of this is also true. If Waldo does something "unwise" then Oswald will gain an advantage simply because we have made him clever by definition. Returning to figure 1, we take up the game matrix again in search of the best strategies for Waldo in his conflict with Oswald, the criminal.

Waldo will be moving across the board (figuratively, of course) in the rows. Oswald, in the same sense, will be moving down in the columns. *Row* number one, in this conflict, represents defense of the bank to Waldo. *Column* number one represents robbing the bank to Oswald. Thus, where Oswald attacks in Column 1 he is going for the bank. To head Oswald off and save the bank Waldo must go to Row 1. Were he to go to Row 2, the melon patch, the criminal would have a free chance at the bank and win 3 points. That is to say, if Waldo is in 4 of Row 1 and Oswald attacks the bank, Oswald will be arrested and Waldo wins 4 points. Likewise, Waldo being in 4 of Row 2 and Oswald attacks the melon patch, Oswald will be arrested and Waldo wins 4 points.

We are now ready to compute Waldo's strategies. The computations are correct, and the reader may accept them as presented without too much concern for their derivation. If further details are desired, the reader is referred to the reference at the end of the text. To compute the strategies we subtract Column 2 from Column 1 in each row. That is, we subtract 1 from 4 in Row 1 and 4 from 3 in Row 2. The results are 3 in Row 1 and −1 in Row 2. Now we switch the rows, which gives us −1 in Row 1 and 3 in Row 2. These are set forth in the right hand column of figure 2.

the sum of Waldo's strategies (1 + 3 = 4), we have $^{13}\!/_4 = 3.25$.

If Waldo's computed strategies always yield the same expected value when played against Oswald then we know that they are the best mixed set we can get.

$$\text{Oswald (1)} \quad \frac{(1 \times 4) + (3 \times 3)}{1 + 3} = \frac{4 + 9}{4} = \frac{13}{4} = 3\tfrac{1}{4}$$

Waldo		Oswald Column 1	Oswald Column 2	Waldo's Strategies
	Row 1	4	1	−1
	Row 2	3	4	3

FIGURE 2

To test these strategies they must be "played" against either of Oswald's moves. Waldo is going to play his strategies (1 and 3) against any single or mixed set of moves by Oswald. Oswald is selecting columns and Waldo is selecting rows. Oswald's first strategy is Column 1 (it could be either 1 or 2 since we are not concerned with what he does). If Oswald plays Column 1 then he stands a chance of winning 3 points or losing 4, depending upon Waldo's defensive more. To compute Waldo's maximum winnings we simply play his strategies (1 and 3) against either outcome (Oswald selects Column 1 or 2). For Oswald's choice of Column 1 the computation (disregarding negative signs) is set forth in figure 3. We then

$$\text{Oswald (2)} \quad \frac{(1 \times 1) + (3 \times 4)}{1 + 3} = \frac{1 + 12}{4} = \frac{13}{4} = 3\tfrac{1}{4}$$

In both cases the yield is 3.25, so we know our strategies are good. The value 3.25 represents the expected winnings of Waldo for each play of the game against Oswald's attempts to capture either the bank or the melon patch.

So we have guaranteed Waldo that if he plays the game according to our computed mixture he will be able to *expect no less* than 3.25 points for each play of the game—on one other condition. This other condition is a simple one. Waldo must select his strategies in a random manner so that Oswald will never be able to predict what he will

Waldo		Oswald Column 1	Oswald Column 2	Waldo's Strategies
	Row 1	4		−1
	Row 2	3		3

$$= \begin{array}{r} 1 \times 4 = 4 \\ 3 \times 3 = \underline{9} \\ 4 + 9 = 13 \end{array}$$

FIGURE 3

divide the value 13 by the sum of the computed strategies (1 + 3 = 4). $^{13}\!/_4 = 3.25$. This gives us the "average" or expected payoff for this strategy.

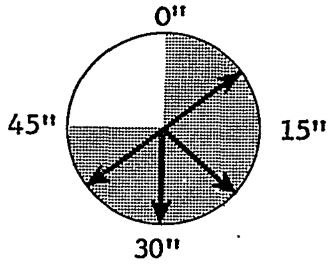
Figure 4 shows the computations for Oswald's choice of Column 2. Again dividing the value by

do. To do this Waldo must devise some means for random action. An easy way would be to look at his watch, and if the sweep second hand is between zero and fifteen seconds to the minute, he should select Row 2. If it is between fifteen seconds to the minute and zero, he should select Row 1. (figure 5).

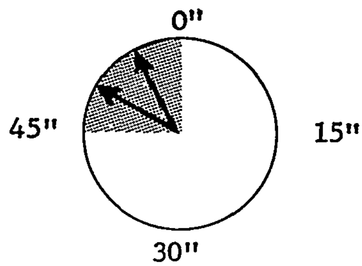
Waldo		Oswald Column 1	Oswald Column 2	Waldo's Strategies
	Row 1		1	1
	Row 2		4	3

$$= \begin{array}{r} 1 \times 1 = 1 \\ 4 \times 3 = \underline{12} \\ 12 + 1 = 13 \end{array}$$

FIGURE 4



Select Row 2
(3 out of 4)



Select Row 1
(1 out of 4)

FIGURE 5

What would happen to Waldo's expected winnings if he decided to ignore his computed strategies, and just try to run alternately from one place to the other? Then his strategy values are equal, i.e. each 1 and figure 6 computes the win-

		Oswald Column 1	Oswald Column 2	Waldo's Strategies
Waldo	Row 1	4	1	1
	Row 2	3	4	1

$$\frac{(1 \times 4) + (1 \times 3)}{1 + 1} = \frac{4 + 3}{2} = \frac{7}{2} = 3\tfrac{1}{2}$$

$$\frac{(1 \times 1) + (1 \times 4)}{1 + 1} = \frac{1 + 4}{2} = \frac{5}{2} = 2\tfrac{1}{2}$$

FIGURE 6

nings. So Waldo would be guaranteed only 2.5 points for each play as against his original 3.25, and in terms of banks and melon patches one can ill afford such a loss!

Now, Waldo has three alternatives open to him for future game playing. (1) He can go to the locker room and sulk because the game he is caught up in seems terribly unfair, since he cannot win all the time. (2) He can accept the fact that in this particular game he is going to lose a little and stick to his strategies, making the criminal suffer as much as possible for each of his depravities. Or, finally, Waldo can cheat. (Not in the moral sense, of course.)

How can Waldo cheat the criminal out of his unjust winnings? In the first place, he can try to get a peek at the "next move" the criminal is contemplating, and at the same time hold his own cards close to his chest. The use of informers, after all, is not a new idea in the "game" of cops and robbers.[3] Waldo should try to gather information about the criminal beyond the basic tabulation of the history of his behavior.

This kind of cheating is accepted in other games, such as Poker. The two games (our example and Poker) are comparable in the sense that in both cases the eventual outcome of the game, *without* the accepted cheating, is determined strictly upon the distribution of either cards or criminals. If, in a Draw Poker game, we were to accept our cards and then draw out the desired number from the remaining deck, paying no attention to the post-deal behavior of the other players, then the status of the game would not change much. However, we do listen carefully to the call for cards as each player draws, and this information adds to our total knowledge of what is going on. We recognize the difference between drawing three cards and drawing none, and this behavior on the part of the other players gives us a small, if not always accurate "peek" at our opponents' hands. So peek when you can, whatever the game.

The game in our example was based upon the assumption of ignorance of all but the past behavior of the criminals. We designated a set of strategies which, under these conditions, would guarantee a maximum of the minimum expected winnings for each play of the game. If Waldo can improve upon the state of his knowledge about the moves of the criminal, then he is going to benefit. If he cannot improve then he is stuck (in this game at least) with a regular expected winning of 3.25.

Randomness in Moves. The strategies computed for the example game, as with all such games, must be selected in a random manner for play. The strategies would be quite worthless unless they were played as a result of being selected by some random device. Just as a strategy is not very good if it can be predicted, so is a policeman not very effective if he can be predicted. The trick is

[3] The term "game" is used here to relate only to the subject and not to the business of professional law enforcement.

to improve your ability to predict the opponent, and at the same time reduce your opponent's ability to predict your moves. The best design is to make your moves in a random manner according to the computed scheme. If you cannot be completely random, then at least make every effort to be unpredictable. If a (beat) officer becomes a predictable agent, then his effectiveness is impaired and he gives advantage to the enemy.

REFERENCE

An exposition and discussion of Game Theory is contained in the book, *The Compleat Strategyst*, by J. D. Williams, published by the McGraw-Hill Company, New York City. The section of the book upon which Random Patrol is based is Chapter 4, pages 132–146. The entire book is on the theory of games of strategy and should be of exceptional value to persons who are not trained in mathematics.